Relations and Functions
Part - 2

ASSERTION AND REASON BASED MCQs (1 Mark each)

Directions : In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false and R is True

Q. 1. Let W be the set of words in the English dictionary. A relation R is defined on W as

$R = \{(x, y) \in W \times W$ such that x and y have at least one letter in common$\}$.

Assertion (A): R is reflexive.

Reason (R): R is symmetric.

Ans. Option (B) is correct.

Explanation: For any word $x \in W$

x and x have atleast one (all) letter in common

$\therefore \quad (x, x) \in R, \forall x \in W \therefore R$ is reflexive

Symmetric: Let $(x, y) \in R, x, y \in W$

$\Rightarrow \quad x$ and y have atleast one letter in common

$\Rightarrow \quad y$ and x have atleast one letter in common

$\Rightarrow \quad (y, x) \in R \therefore R$ is symmetric

Hence A is true, R is true; R is not a correct explanation for A.

Q. 2. Let R be the relation in the set of integers Z given by $R = \{(a, b) : 2$ divides $a - b\}$.

Assertion (A): R is a reflexive relation.

Reason (R): A relation is said to be reflexive if xRx, $\forall x \in Z$.

Ans. Option (A) is correct.

Explanation: By definition, a relation in Z is said to be reflexive if xRx, $\forall x \in Z$. So R is true.

$a - a = 0 \Rightarrow 2$ divides $a - a \Rightarrow aRa$.

Hence R is reflexive and A is true.

R is the correct explanation for A.

Q. 3. Consider the set $A = \{1, 3, 5\}$.

Assertion (A): The number of reflexive relations on set A is 2^9.

Reason (R): A relation is said to be reflexive if xRx, $\forall x \in A$.

Ans. Option (D) is correct.

Explanation: By definition, a relation in A is said to be reflexive if xRx, $\forall\, x \in A$. So R is true.

The number of reflexive relations on a set containing n elements is 2^{n^2-n}.

Here $n = 3$.

The number of reflexive relations on a set $A = 2^{9-3} = 2^6$.

Hence A is false.

Q. 4. Consider the function $f : R \to R$ defined as $f(x) = x^3$

Assertion (A): $f(x)$ is a one-one function.

Reason (R): $f(x)$ is a one-one function if co-domain = range.

Ans. Option (C) is correct.

Explanation: $f(x)$ is a one-one function if

$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2.$$

Hence R is false.

Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in R$

$\Rightarrow \quad (x_1)^3 = (x_2)^3$

$\Rightarrow \quad x_1 = x_2$

Hence $f(x)$ is one-one.

Hence A is true.

Q. 5. If $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and $f = \{(1, 4), (2, 5), (3, 6)\}$ is a function from A to B.

Assertion (A): $f(x)$ is a one-one function.

Reason (R): $f(x)$ is an onto function.

Ans. Option (C) is correct.

Given, $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and $f : A \to B$ is defined as $f = \{(1, 4), (2, 5), (3, 6)\}$ *i.e.*, $f(1) = 4$, $f(2) = 5$ and $f(3) = 6$.

It can be seen that the images of distinct elements of A under f are distinct. So, f is one-one.

So, A is true.

Range of $f = \{4, 5, 6\}$.

Co-domain $= \{4, 5, 6, 7\}$.

Since co-domain $\neq$ range, $f(x)$ is not an onto function. Hence R is false.

Q. 6. Consider the function $f : R \to R$ defined as

$$f(x) = \frac{x}{x^2+1}.$$

Assertion (A): $f(x)$ is not one-one.

Reason (R): $f(x)$ is not onto.

Ans. Option (B) is correct.

Explanation: Given, $f : R \to R$;

$$f(x) = \frac{x}{1+x^2}$$

Taking $x_1 = 4$, $x_2 = \frac{1}{4} \in R$

$$f(x_1) = f(4) = \frac{4}{17}$$

$$f(x_2) = f\left(\frac{1}{4}\right) = \frac{4}{17} \qquad (x_1 \neq x_2)$$

$\therefore$ f is not one-one.

A is true.

Let $y \in R$ (co-domain)

$f(x) = y$

$$\Rightarrow \quad \frac{x}{1+x^2} = y$$

$$\Rightarrow \quad y.(1 + x^2) = x$$

$$\Rightarrow \quad yx^2 + y - x = 0$$

$$\Rightarrow \quad x = \frac{1 \pm \sqrt{1-4y^2}}{2y}$$

since, $x \in R$,

$$\therefore \quad 1 - 4y^2 \geq 0$$

$$\Rightarrow \quad -\frac{1}{2} \leq y \leq \frac{1}{2}$$

So Range $(f) \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

Range $(f) \neq R$ (Co-domain)

$\therefore f$ is not onto.

R is true.

R is not the correct explanation for A.